Expansion of the Universe and redshift

Primo Galletti Aldo Aluigi

Rome, 21 September 2002

In a Universe where *matter continuously born and dies* [1] it is no more possible to give support to the hypothesis of an initial big explosion (Big-Bang), which happened about 20 billion years ago, to explain its expansion [2].

We intend to propose here a different interpretation of the expansion of the Universe that is more in line with the existence of a "physical" space.

We will see how with this new interpretation the meaning of *redshift* z results very different from the Doppler effect, even if the corresponding expression is nearly to the following [3]:

$$z = \frac{T_B - T_A}{T_A}$$

that, as very well known, matches one of the expressions of the "classical" Doppler effect [4]:

$$z + 1 = \frac{c}{c - v} \qquad (1)$$

[1]See in this respect **Part 2** and **5** of *A detector for Gravitational Waves*.

[2]Through Big-Bang, redshift z is interpreted in terms of ("relativistic") Doppler effect of the electromagnetic radiation emitted by celestial objects going away from us. To be more precise:

$$z + 1 = \sqrt{\frac{c + v}{c - v}}$$

We only want to stress how this small distances, practically, coincides with (1) ($z \leq 0.5$) (see **Figure 7**).

There is to notice that the validity of the above expression has been verified only through the analysis of electromagnetic radiations emitted by celestial bodies quite near us.

[3]Let us consider A and B two celestial bodies that we suppose are getting away from each other at speed v. If at time $t = t_0$ body A emits a signal lasting T_A which can be received by B and we indicate as T_B the time of the signal recorded by this latter one, it results that:

$$T_A > T_B$$

This effect is usually called redshift as it consists of a shifting towards low frequencies (that is to say towards red, in case the signal is a luminous one) of the signal spectrum recorded by B and it is defined as follows:

[4]In **Appendix A.1** expressions concerning the "classical" Doppler effect are reported.

We have already seen how (1) work well in the analysis of high intensity gravitational waves ("forks"), caused by collapsing of nuclei of *Multiple Nucleus Quasars* (MNQs), even if this happens at very long distance from us [5].

1 Expansion of the Universe

We intend to work out the expression having redshift and the other parameters related to it, in the hypothesis that *the expansion of the Universe is due to a continuous interposition (generation) of new "physical" space.*

In this aim, Γ_0 is indicated as the rate of "physical" space generated in a unit of time per unit of volume [6]:

$$\Gamma_0 = m^3/s \; of \; space \; per \; m^3 \; of \; volume$$

Let us suppose A and B be two celestial bodies and consider a sphere having its centre in A and the radius corresponding to r_{AB} so that on its surface body B is contained (see **Figure 1**).

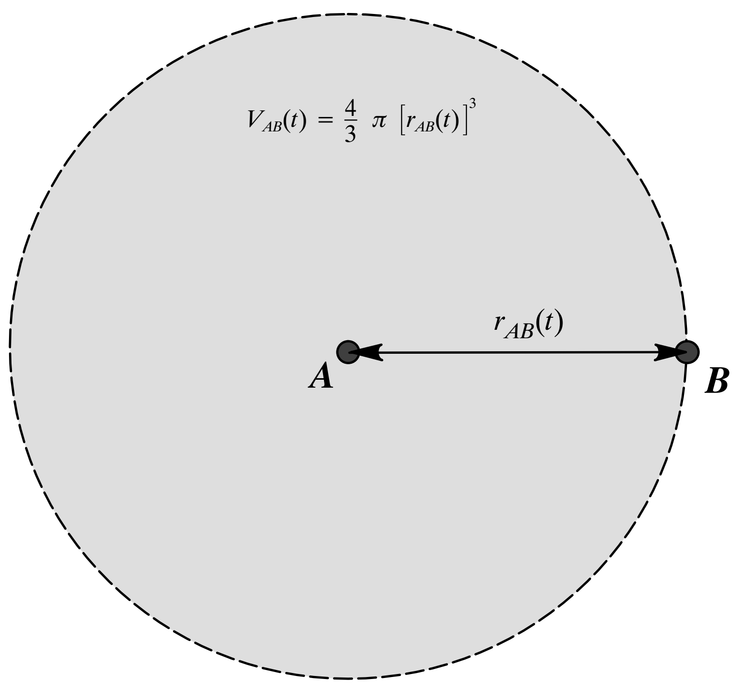

Figure 1: Moving away of A and B due to the expansion process

[5] See **Appendix A.1** concerning **Part 2** of *A detector for Gravitational Waves.*

[6] By now, we are not going to deal about the way such media may be generated.

Volume variation $\Delta V_{AB}(t)$ per unit of time of the sphere, built in this way, is directly proportional to the volume $V_{AB}(t)$ itself. And, more exactly, for volume V_{AB}, it is possible to write the following relation (differential):

$$\dot{V}_{AB} = \Gamma_0 \ V_{AB} \tag{2}$$

where a dot represents the derivative in respect with time. The solution of (8), supposing that Γ_0 may be considered as constant, is given by:

$$V_{AB}(t) = V_0 \ e^{\Gamma_0 \ (t-t_0)} \tag{3}$$

where V_0 is the initial volume ($t = t_0$). It is easy, on the contrary, to verify that for the distance $r_{AB}(t)$ the following relation is effective:

$$r_{AB}(t) = r_0 \ e^{\frac{\Gamma_0}{3} \ (t-t_0)} \tag{4}$$

where r_0 is the initial distance between A and B ($t = t_0$). If we take the derivative of (4) in respect with time, we obtain v_{AB} indicating the getting away speed between the two bodies:

$$v_{AB} \equiv \dot{r}_{AB} = \frac{\Gamma_0}{3} \ r_{AB} \tag{5}$$

(5) shows the existence of proportionality between the getting away speed and distance.

What above specified can be summarized as follows: *in as much as the rate of space production* Γ_0 *may be considered as constant, the two bodies A and B move away at a speed which is directly proportional to their distance.*

If we compare (5) with Hubble's law:

$$v = H_0 \ r \tag{6}$$

we obtain the relation existing between Γ_0 and the Hubble constant H_0 [7]:

$$\Gamma_0 = 3 \ H_0 \tag{7}$$

for which reason (3) and(4) can be also written in the following way:

$$V_{AB}(t) = V_0 \ e^{3 \ H_0 \ (t-t_0)} \tag{8}$$

$$r_{AB}(t) = r_0 \ e^{H_0 \ (t-t_0)} \tag{9}$$

2 Signals propagation

Let us now suppose that A emits a pulse signal that can be received by B. Let r_0 be the distance between A and B at the time t_0 when the pulse is emitted. We

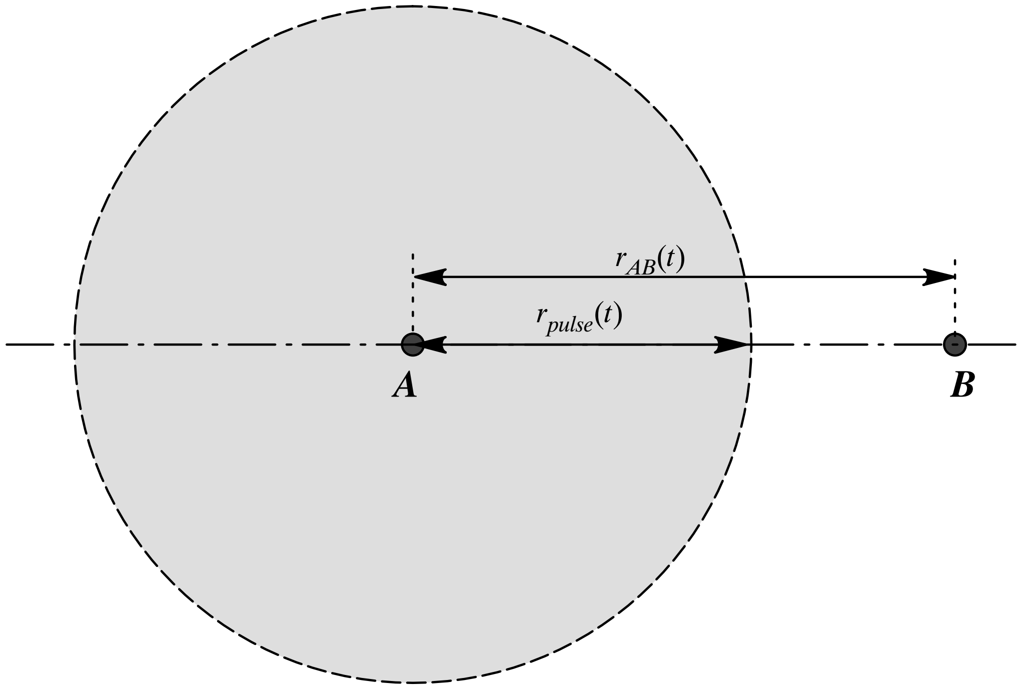

Figure 2: Propagation of the pulse emitted by A

also indicate as $r_{pulse}(t)$ distance, for $t > t_0$, covered by the impulse (see **Figure 2**).

For $r_{pulse}(t)$ we can write the following (differential) relation:

$$\dot{r}_{pulse}(t) = c + v_{pulse}(t) \tag{10}$$

where, dove, c is the propagation speed of the signal (in respect to the medium of communication) and $v_{pulse} \equiv \dot{r}_{pulse}$ is the expansion speed of the sphere having radius r_{pulse}. As due to (5) we have:

$$v_{pulse}(t) = \frac{\Gamma_0}{3}\ r_{pulse}(t) \equiv H_0\ r_{pulse}(t) \tag{11}$$

If we put (11) into (10) we obtain the following (differential) expression:

$$\frac{dr_{pulse}}{c + H_0\ r_{pulse}} = dt \tag{12}$$

and this, if integrated with the initial condition:

$$r_{pulse}(t_0) = 0$$

[7]There is still no agreement about the value to be attributed to Hubble constant. Its value seems to be between 45 and 80 km/s by Mpc (that is to say between 12.5 and 25 km/s by million of light-year).

gives the following:

$$\ln \frac{c + H_0\ r_{pulse}}{c} = H_0\ (t - t_0) \tag{13}$$

from which, we obtain:

$$r_{pulse}(t) = \frac{c}{H_0}\ \left[e^{H_0\ (t-t_0)} - 1\right] \tag{14}$$

It is possible to calculate time Δt needed by the impulse before being received by B making (14) equal to (9). We obtain the following very important expression:

$$\Delta t = \frac{1}{H_0}\ \ln \frac{c}{c - H_0\ r_0} \equiv \frac{1}{H_0}\ \ln \frac{c}{c - v_0} \tag{15}$$

where v_0 is the initial speed between A and B (that is to say at the moment when A emits the signal):

$$v_0 = H_0\ r_{AB}(t_0) = H_0\ r_0 \tag{16}$$

The distance between A and B at the moment when the signal is received by B is obtained by placing (15) into (9):

$$r_{AB}(\Delta t) = r_0\ \frac{R_U}{R_U - r_0} \tag{17}$$

where r_0 is the initial distance between A and B. The distance covered by the impulse before being received by B can be obtained placing (15) into (14):

$$r_{pulse}(\Delta t) = R_U\ \frac{r_0}{R_U - r_0} \tag{18}$$

3 Radius of the visible Universe

If we observe (15), (17) and (18) we notice that there are finite travelling times for the signal if:

$$r_0 < r_{crit} \equiv \frac{c}{H_0} \tag{19}$$

that is to say if, at the moment when the signal was emitted, source A was at a distance of receiver B shorter than the critical distance r_{crit}.

We can, therefore, define the *Radius of the Visible Universe* using the following quantity:

$$R_U = \frac{3\ c}{\Gamma_0} \equiv \frac{c}{H_0} \tag{20}$$

It is easy to verify that R_U is a specified distance between A and B where the getting away speed v becomes equal to the speed of impulse c. Namely:

$$v = H_0 \; R_U = H_0 \; \frac{c}{H_0} = c \tag{21}$$

For distances between A and B equal to or higher than R_U the impulses emitted by A cannot reach B any more (that is to say, body A cannot be "seen" any more by B).

Definition (19), allows to write (15) also in the following way:

$$\Delta t = t_H \;\; \ln \frac{R_U}{R_U - r_0} \tag{22}$$

where,

$$t_H \equiv \frac{1}{H_0} = \frac{3}{\Gamma_0} \tag{23}$$

represent the *characteristic time (Hubble's time)* of the Universe expansion process. Namely, if in (14) we put:

$$t - t_0 = t_H$$

we obtain that the impulse has covered a distance corresponding to $(1 - e) \; R_U$ (that is to say about 63 % of R_U) or, still better, t_H is the time necessary to reach a speed to get away between A and B corresponding to about 63 % of c.

Time t_2 to double the starting distance (r_0) between A and B is obtained in making equal to 2 r_0 the expression (9):

$$2 \; r_0 = r_0 \; e^{H_0 \;\, t_2} \tag{24}$$

and out of this we obtain:

$$t_2 = \frac{\ln 2}{H_0} = 0.693... \; t_H \tag{25}$$

t_2 is also the time necessary in order that the initial volume (V_0) becomes 8 times bigger.

4 A geometrical representation

It is possible to give a very simple geometrical interpretation of what above said. If we put:

$$w = e^{H_0 \;\, (t - t_0)} \tag{26}$$

both (9) and (14) become linear functions of the new variable w (see **Figure 3**).

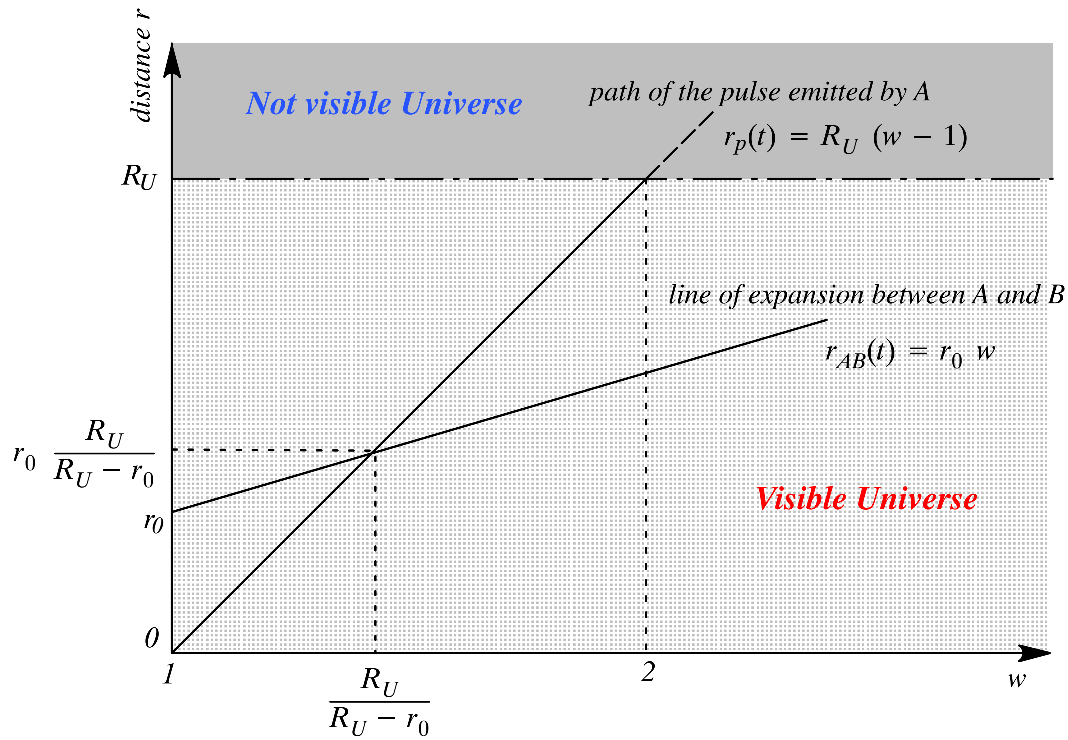

Figure 3: Geometrical representation of the pulse emitted by A

It is easy to verify that the intersection of the two straight lines represents the instant when the pulse emited by A is received by B. The intersection of the two lines gives the following:

$$w = \frac{R_U}{R_U - r_0} \tag{27}$$

$$r = r_0\ \frac{R_U}{R_U - r_0} \tag{28}$$

that, along with (20), as we will see represent the main relations of the Universe expansion process.

5 Redshift and wave's widening

Let us suppose that at time $t = t_0$ the celestial body A emits pulse 1 and after a given time T_A emits pulse 2. Let us consider, instead, T_B as the time interval elapsing, during which B receives impulse 1 and impulse 2 (see **Figure 4**).

According to the hypotheses stated in the previous points we want now to bring out the relation existing between T_A and T_B . If we consider as $r_1(t)$ and $r_2(t)$ the two distances from A respectively covered by pulse 1 and pulse 2 (see **Figure 5**).

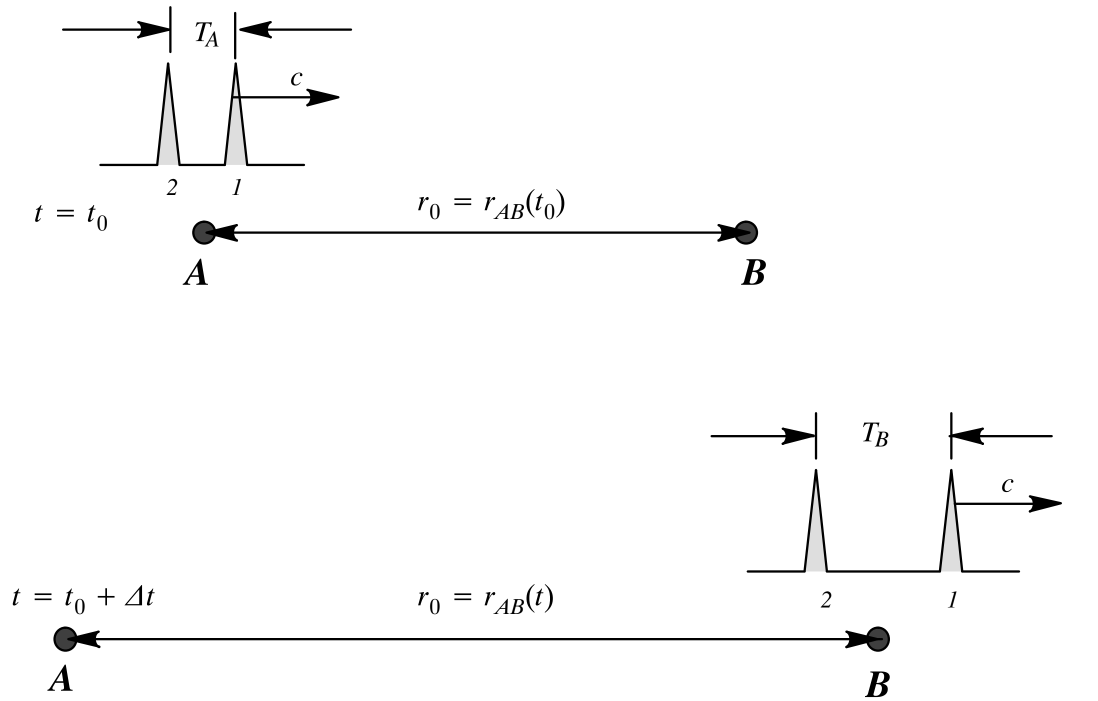

Figure 4: Propagation of the couple of pulses emitted by A

Travelling time of pulse 1, on the basis of (22), is given by:

$$\Delta t_1 = \frac{1}{H_0} \ \ln \frac{R_U}{R_U - r_0} \tag{29}$$

where r_0 is the initial distance between A and B. For pulse 2, on the contrary, we have:

$$\Delta t_2 = \frac{1}{H_0} \ \ln \frac{R_U}{R_U - \big(r_0 + \Delta r_0\big)} \tag{30}$$

where Δr_0 is the increase of the distance between A and B during the emission of the two pulses. If we use (9) we have:

$$r_0 + \Delta r_0 = r_0 \ e^{H_0 \ T_A}$$

And as,

$$H_0 \ T_A \ll 1$$

we have, with a very good approximation:

$$\Delta r_0 \approx r_0 \ H_0 \ T_A$$

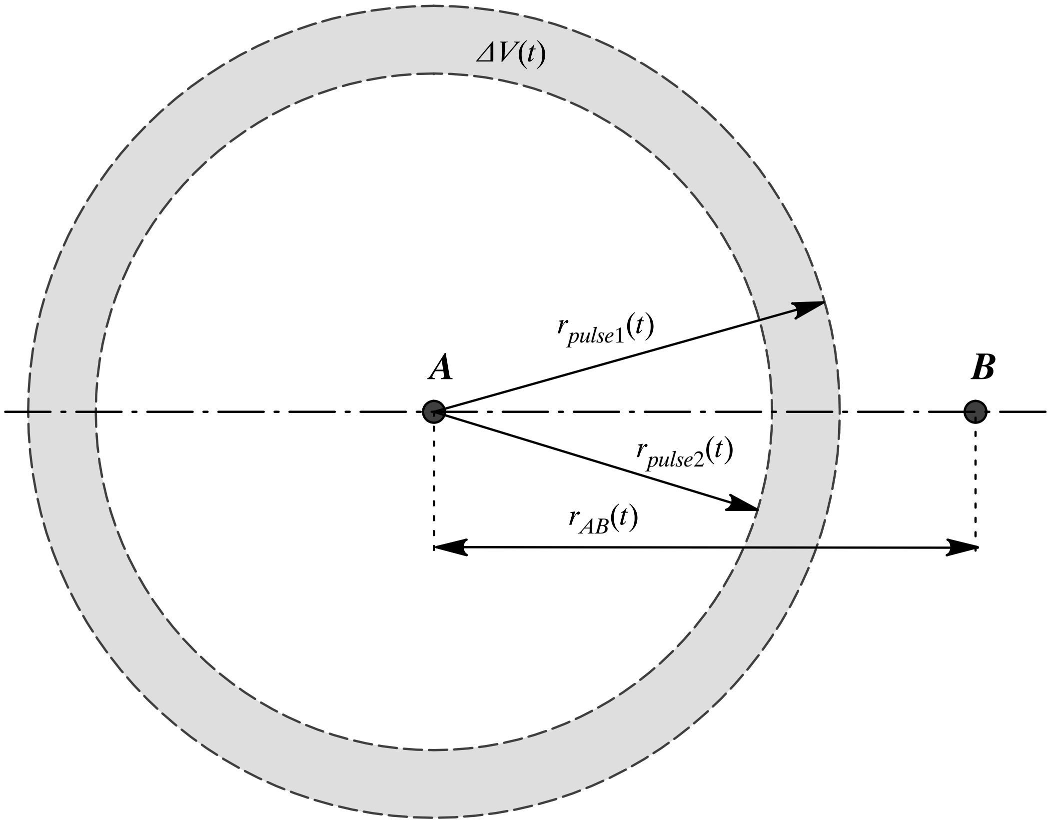

Figure 5: Widening of the couple of pulses emitted by A

Now, time T_B between reception of the two pulses by B is given by:

$$T_B = T_A + \Delta t_2 - \Delta t_1 \tag{31}$$

where, if we replace (29) and (30), we obtain:

$$T_B = T_A + \Delta t_2 - \Delta t_1 = T_A - \frac{1}{H_0} \ \ln \frac{R_U - \left(r_0 - \Delta r_0\right)}{R_U - r_0} \tag{32}$$

Furthermore, as,

$$\frac{1}{H_0} \ \ln\left(1 - \frac{r_0}{R_U - r_0} \ H_0 \ T_A\right) \approx -T_A \ \frac{r_0}{R_U - r_0} \tag{33}$$

(32) is simplified and becomes::

$$T_B = T_A - T_A \ \frac{r_0}{R_U - r_0} = T_A \ \frac{R_U}{R_U - r_0} \equiv T_A \ \frac{c}{c - v_0} \tag{34}$$

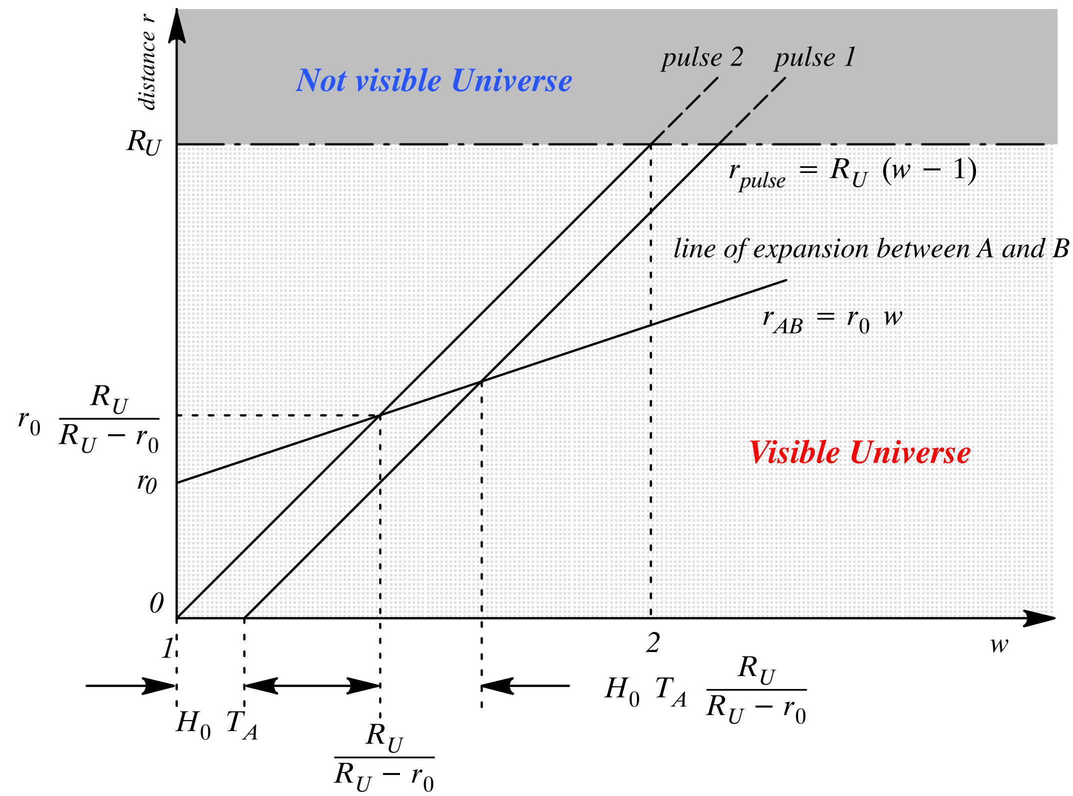

Figure 6: Geometrical representation of the couple of pulses emitted by A

(34) represents the relation for between time interval T_A that is the emission of impulses by A and time interval T_B that is the reception of same by B. **Figure 6** illustrates the situation we have in this case.

If we introduce the *redshift* z:

$$z = \frac{T_B - T_A}{T_A} \tag{35}$$

(34) becomes:

$$T_B = T_A\ (z + 1) \tag{36}$$

so we obtain:

$$z = \frac{v_0}{c - v_0} \equiv \frac{r_0}{R_U - r_0} \tag{37}$$

It is easy to verify that variable w, previously introduced, is nothing but the widening between the two impulses due to an expansion process:

$$w = \frac{T_B}{T_A} \tag{38}$$

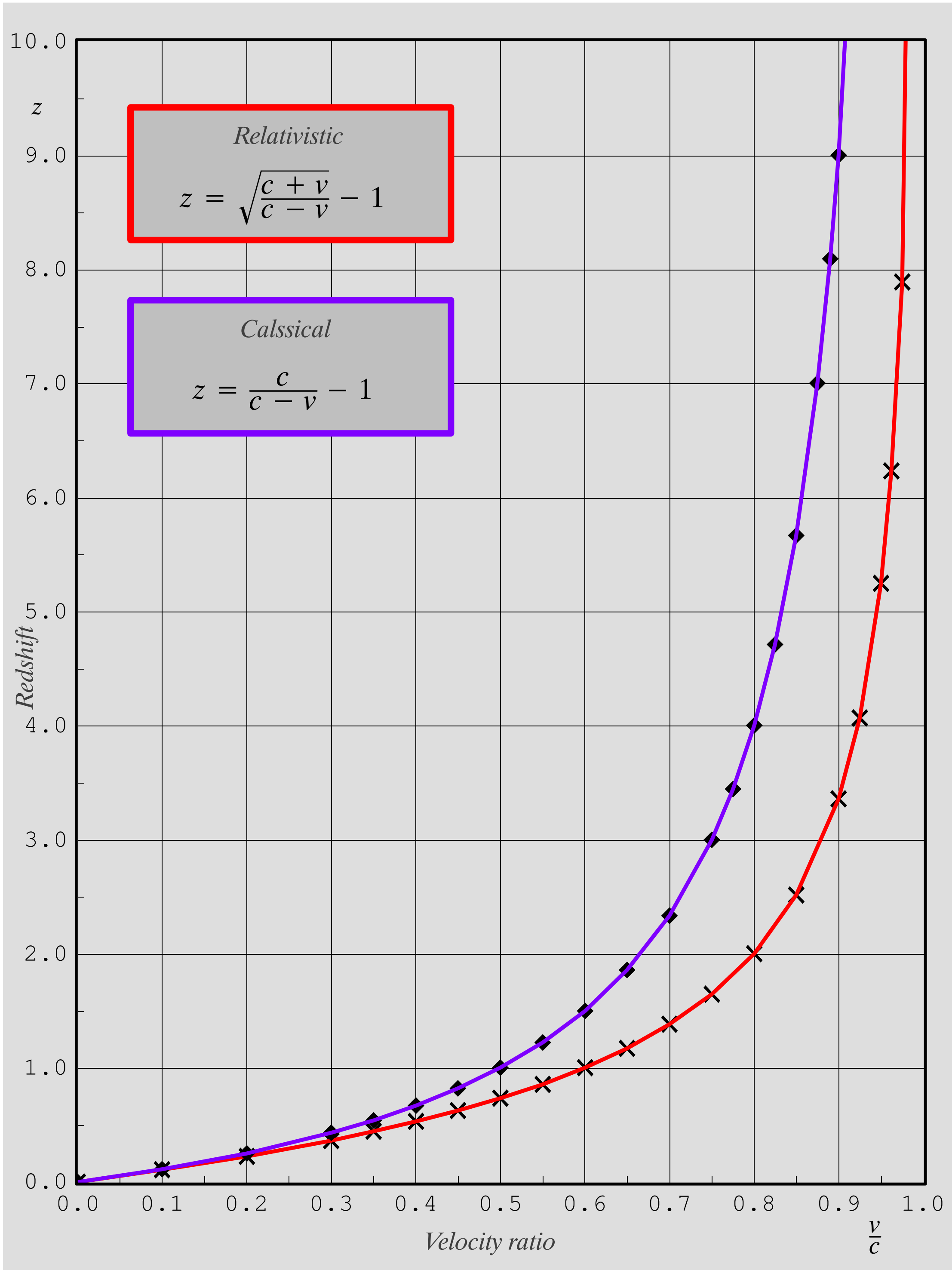

Figure 7: Differences between "classical" and "relativistic" redshift

and namely,

$$w = \frac{c}{c - v_0} \equiv \frac{R_U}{R_U - r_0} \tag{39}$$

In combining (37) with (39) we can obtain the relation between redshift z and widening w:

$$w = z + 1 \tag{40}$$

And thus, previous relations (9) and (15) become:

$$r_{AB} = r_0 \ (z + 1) \tag{41}$$

$$\Delta t = \frac{1}{H_0} \ \ln(z + 1) \tag{42}$$

For volume V_{AB}, the following relation become effective:

$$V_{AB} = V_{=} \ (z + 1)^3 \tag{43}$$

and a relationship of the same kind is, as well, is effective for volume ΔV between the two impulses.

Given as known redshift z (or widening w and propagation speed c, it is possible to calculate the initial getting away speed v_0 of the object. Namely, from (37) and (39) we obtain:

$$v_0 = c \ \frac{z}{z + 1} \tag{44}$$

As in this case the getting away speed is given by:

$$v_0 = H_0 \ r_0 \tag{45}$$

If we compare (45) with (44) we will obtain:

$$r_0 = R_U \ \frac{z}{z + 1} \tag{46}$$

Therefore, if the radius of the visible Universe R_U is known, it is possible to calculate the distance from us to the object we are discussing about.

And finally, if we compare (44) with (46) we can notice that the speed and the distance of the object have the same dependence from redshift.

6 Discussion

With this *new interpretation of the Universe expansion as continuous generation of "physical" space*, the following remarks have to be considered:

1. *Redshift is substantially distinguished from the Doppler effect* (even if the expression obtained is mainly the same). Here, both A and B are always in a state at rest respect to communication medium surrounding them.

2. *There is an exact symmetry between source A and receiver B.* That is to say, the expression for redshift does not change if it is B that emits the pulses and it is A that receives them.

3. There exists a *critical distance* beyond which the phenomenon has no more its physical meaning. Said distance has been called as *Radius of the visible Universe.*

4. It is possible to talk of radius of visible Universe as long as Γ_0 (therefore, H_0 as well), can be considered constant.

5. Both expansion speed between the two bodies A and B and their distance have the same dependence from redshift z.

6. w represents widening in time of the impulse which, as we have seen, also results as its widening in space.

7. Widening w, on a physical point of view, is a more appropriate parameter than redshift z to represent the process of expansion.

To conclude, we want to mention "calibration" problem of the redshift curve. This is, substantially, how to determinate the radius of the visible Universe (therefore of Hubble constant), which, if known, once widening w (or redshift z) is given, allows to calculate how distant from us is the celestial body that emitted the pulse.

It is well known, that to be able to calibrate this curve it is necessary to find out a *"standard candle"*, that is to say, a source whose characteristics for what concerns energy are known "a priori".

We will discuss about these problems later on, and try to use the phenomenon of the collapsing of MNQs as "standard candle".

A APPENDIX

A.1 "Classical" Doppler effect

If we take a source S emitting impulses having their own period T_S and a receiver R receiving the impulses emitted by S. We want to state the period of impulses T_R received by R. The medium between S ed R has propagation speed of the impulses c.

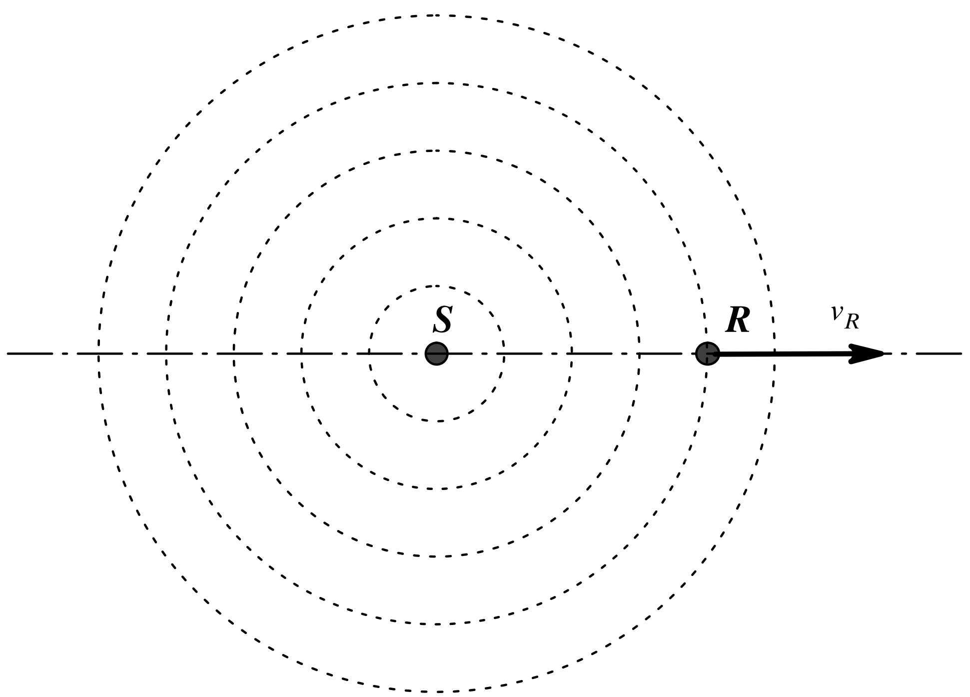

Figure 8: Source (S) at rest and receiver (R) moving

Case 1: Source (S) at rest and receiver (R) moving.

If R is going away at speed v_R, reception periodo T_R of the impulses is calculated according to the following expression:

$$T_R = T_S + \frac{\Delta r_R}{c} \tag{47}$$

where Δr_R is the distance covered by R between the arrival of the two impulses. As,

$$\Delta r_R = v_R \; T_R \tag{48}$$

We are going to replace (48) into (47) and in obtaining T_R we have:

$$T_R = T_S \; \frac{c}{c - v_R} \tag{49}$$

That is to say, R receives impulses having a longer period than the emitted one (see **Figure 8**).

On the contrary, if R is approaching, it is easy to verify that:

$$T_R = T_S \; \frac{c}{c + v_R} \tag{50}$$

Case 2: Source (S) moving and receiver (R) at rest.

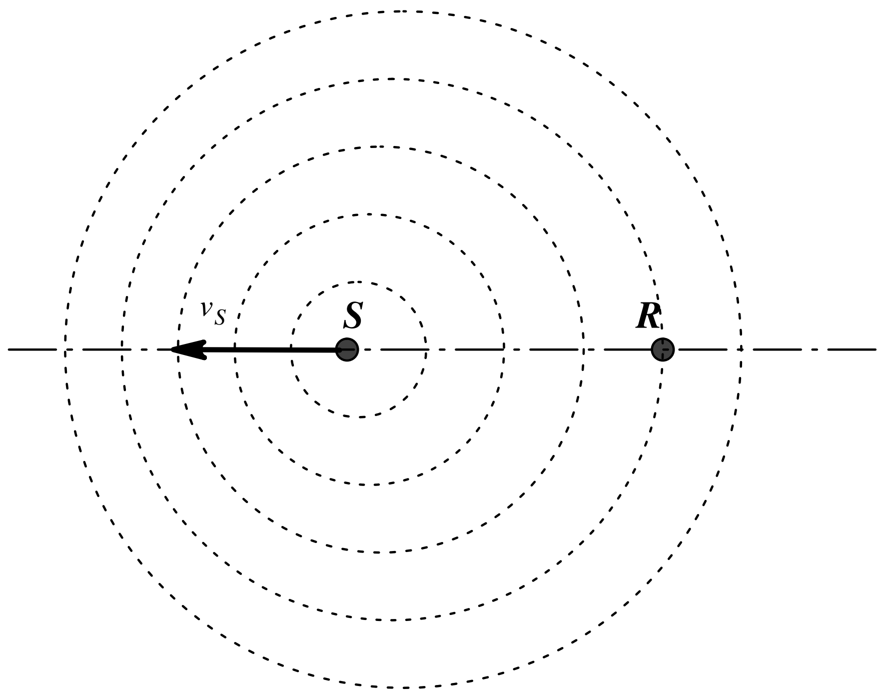

Figure 9: Receiver (R) at rest and source (S) moving

In this case R sees source S emitting impulses having a supposed (apparent) period given by the following expression:

$$T_R = T_S + \frac{\Delta r_S}{c} \tag{51}$$

where Δr_S is the distance covered by S between the two emitted impulses. As in this case too it comes out that:

$$\Delta r_S = v_S \; T_S \tag{52}$$

If we replace (52) with (51) we obtain:

$$T_R = T_S \, \frac{c + v_S}{c} \tag{53}$$

Figure 9 shows what we obtain in this case.

While if, on the contrary, if S is approaching, we have:

$$T_R \equiv T'_S = T_S \, \frac{c - v_S}{c} \tag{54}$$

In comparing (49) and (50) with (53) and (54), it is possible to notice that the effect due to the movement of R is *different* from the effect due to the movement of S.